All the conic sections we studied in this unit had horizontal and/or vertical axes (none of them were "tilted").

All such conic section equations can be written in the form

$$\boldsymbol{Ax^2 + By^2 + Cx + Dy + E = 0}$$

or

$$\boldsymbol{Ay^2 + Bx^2 + Cx + Dy + E = 0}$$

where $A, B, C, D, \& E$ are all integers (possibly including 0) and A is non-negative.

We have learned about four conic sections: circles, ellipses, parabolas, & hyperbolas. Let's look at the differences and similarities of their equations.

Circles

$$(x-h)^2+(y-k)^2=r^2$$

1) **Both** x^2 & y^2 terms appear in a circle equation
2) The x^2 & y^2 terms are connected by a $+$ sign.
3) The coefficients of x^2 & y^2 are **the same** ($A=B$, and both are positive) when a circle is written as $Ax^2+By^2+Cx+Dy+E=0$.

Ellipses

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

1) **Both** x^2 & y^2 terms appear in an ellipse equation
2) The x^2 & y^2 terms are connected by a $+$ sign.
3) The coefficients of x^2 & y^2 are **NOT the same** ($A\neq B$, though both are positive) when an ellipse is written as $x^2+By^2+Cx+Dy+E=0$.

Hyperbolas

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \quad \text{or} \quad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

1) **Both** x^2 & y^2 terms appear in a hyperbola equation
2) The x^2 & y^2 terms are connected by a $-$ sign.
3) **One** of the coefficients of x^2 & y^2 **is positive** and the **other is negative** (A is positive & B is negative) when a hyperbola is written as $Ax^2+By^2+Cx+Dy+E=0$ (or $Ay^2+Bx^2+Cx+Dy+E=0$).

Parabolas

$$y-k=a(x-h)^2 \quad \text{or} \quad x-h=a(y-k)^2$$

1) **Exactly one** x^2 or y^2 term appears in a parabola equation – **NEVER BOTH**!
2) Either $A=0$ or $B=0$ when a parabola is written as $Ax^2+By^2+Cx+Dy+E=0$.

Example 1

Is the following equation for a circle, ellipse, hyperbola, or parabola?

$$4x^2 - 9y^2 + 32x - 144y - 548 = 0$$

Start asking these questions:

1) Does this equation have **both** x^2 & y^2 terms?
   Yes (that rules out parabolas)
2) Are they **both positive** or is **one positive/one negative**?
   One positive/one negative (that rules out circles and ellipses)

It is a HYPERBOLA

Example 2

Is the following equation for a circle, ellipse, hyperbola, or parabola?

$$2x^2 + y^2 - 4x - 4 = 0$$

Start asking these questions:

1) Does this equation have **both** x^2 & y^2 terms?
   Yes (that rules out parabolas)
2) Are they **both positive** or is **one positive/one negative**?
   Both positive (that rules hyperbolas)
3) Are the coefficients of the x^2 & y^2 terms **equal** or **not equal**?
   Not equal (that rules out circles)

It is an ELLIPSE

Example 3

Is the following equation for a circle, ellipse, hyperbola, or parabola?

$$x^2 + 4x - 8y + 12 = 0$$

Start asking these questions:

1) Does this equation have **both** x^2 & y^2 terms?
   No – just an x^2 term (that rules out circles, ellipses, and hyperbolas)

It is a PARABOLA

Example 4

Is the following equation for a circle, ellipse, hyperbola, or parabola?

$$x^2 + y^2 - 6x + 14y - 60 = 0$$

Start asking these questions:

1) Does this equation have **both** x^2 & y^2 terms?
   Yes (that rules out parabolas)
2) Are they **both positive** or is **one positive/one negative**?
   Both positive (that rules hyperbolas)
3) Are the coefficients of the x^2 & y^2 terms **equal** or **not equal**?
   Equal (that rules out ellipses)

It is a CIRCLE